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Polynomial Inverse of Kellerian Mapping

Dang Vu Giang

Hanoi Institute of Mathematics

18 Hoang Quoc Viet, 10307 Hanoi, Vietnam

e-mail: ⟨dangvugiang@yahoo.com⟩

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Abstract. Using the uniqueness of orthonormal polynomial basis of the Bergman space over a ball in complex vector space we prove that every Kellerian mapping has polynomial inverse. The Jacobian conjecture is simply proved.

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AMS subject classification: 32M15, 43A85

Kellerian mapping is a polynomial mapping $\Phi : \mathbb{C}^n \to \mathbb{C}^n$ whose Jacobian deternimant is 1. The famous Jacobian conjecture [1] [4] says that the mapping Φ has polynomial inverse. Thousands of authors are interested in proving this conjecture [1] but unsuccesful for more than 70 years. In this paper we use a completely new idea from the orthonormal polynomial basis of a Bergman space [2] [3]. In fact, there is $r > 0$ such that the restriction of Φ into the open ball $B(0, r)$ is injective. Let $\{\varphi_k\}_{k=1}^{\infty}$ be the complete system of orthonormal polynomials of several variables over the body $\Phi(B(0, r))$ with respect to the Lebesgue measure in $\mathbb{R}^{2n} = \mathbb{C}^n$. Then, $\{\varphi_k \circ \Phi\}_{k=1}^{\infty}$ is an orthonormal polynomial basis of the Bergman space over the ball $B(0, r)$. But the system $\{z_1^{\alpha_1} z_2^{\alpha_2} \cdots z_n^{\alpha_n}\}_{\alpha_j=0}^{\infty}$ is the only orthogonal polynomial basis

of this Bergman space. Therefore, there are n polynomials φ_{k_j} from the basis $\{\varphi_k\}_{k=1}^{\infty}$ such that

$$z_j = \kappa_j \varphi_{k_j} \circ \Phi\left(z_1, z_2, \cdots, z_n\right)$$

for every $j = 1, 2, \cdots, n$. This means exactly, Φ has polynomial inverse. The famous Jacobian conjecture is true.

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